TWO-STREAM INSTABILITY OF COUNTER-ROTATING GALAXIES

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ABSTRACT

The present study of the two-stream instability in stellar disks with counter-rotating components of stars and/or gas is stimulated by recently discovered counter-rotating spiral and S0 galaxies. Strong linear two-stream instability of tightly-wrapped spiral waves is found for one and two-armed waves with the pattern angular speed of the unstable waves always intermediate between the angular speed of the co-rotating matter ($+\Omega$) and that of the counter-rotating matter ($-\Omega$). The instability arises from the interaction of positive and negative energy modes in the co- and counter-rotating components. The unstable waves are in general convective - they move in radius and radial wavenumber space - with the result that amplification of the advected wave is more important than the local growth rate. For a galaxy of co-rotating stars and counter-rotating stars of mass-fraction $\xi_* < \frac{1}{2}$, or of counter-rotating gas of mass-fraction $\xi_g < \frac{1}{2}$, the largest amplification is usually for the one-armed leading waves (with respect to the co-rotating stars). For the case of both counter-rotating stars and gas, the largest amplifications are for $\xi_* + \xi_g \approx \frac{1}{2}$, also for one-armed leading waves. The two-armed trailing waves usually have smaller amplifications. The growth rates and amplifications all decrease as the velocity spreads of the stars and/or gas increase. It is suggested that the spiral waves can provide an effective viscosity for the gas causing its accretion.

1. INTRODUCTION

A surprise of recent high spectral resolution studies of normal galaxies is the occurrence of counter-rotating gas and/or stars in galaxies of all morphological types, ellipticals, spirals, and irregulars (see review by Galletta 1996). In the ellipticals, the counter-rotating component is usually in the nuclear core and may result from merging of galaxies with opposite angular momentum. In contrast, in a number of spiral and $S0$ galaxies, counter-rotating disks of gas and/or stars have been found to co-exist with the primary disk out to large distances ($10-20$ kpc). Examples include NGC 4550 (Rubin, Graham, & Kenney 1992; Rix et al. 1992), NGC 4826 (Braun et al. 1992), NGC 7217 (Merrifield & Kuijken 1994), NGC 4546 (Sage & Galetta 1994), NGC 3626 (Ciri et al. 1995), NGC 3593 (Bertola et al. 1996), and NGC 4138 (Jore, Broeils, & Haynes 1996). Table 1 summarizes data on these galaxies. These galaxies are early type and have ring(s) or dust lanes or other morphological peculiarities. Related cases include galaxies with infalling HI gas, for example, NGC 4254 which has an $m = 1$ spiral arm (Phookun, Vogel, & Mundy 1993), and NGC 628 which has a warped and distorted outer disk with high-velocity clouds (Kamphius & Briggs 1992).

It is not likely that the large-scale counter-rotating components result from mergers of similar mass galaxies of opposite angular momenta because of the large vertical thickening observed in simulation studies of such mergers (Barnes 1992). Formation of disk galaxies by accretion has been studied by Ryden and Gunn (1987) and Ryden (1988). Quinn and Binney (1992) have shown that the more recently accreted matter at large distances has a spin anticorrelated with the spin of the central mattter. Thakar and Ryden (1996) discuss different possibilities for the formation of counter-rotating galaxies, (1) that the counter-rotating matter may come from the merger of an oppositely rotating gas rich dwarf galaxy with an existing spiral, and (2) that the accretion of gas may occur over the lifetime of a galaxy with the more recently accreted gas counter-rotating.

An important open theoretical question is: What is the interaction between the co- and counter-rotating components observed on large-scales in spiral and $S0$ galaxies, and does this interaction facilitate accretion of the counter-rotating component? The component without neutral or ionized gas rotating in the same direction is referred to as the co-rotating component. Clearly, a strong interaction will occur between co- and counter-rotating gas with the result that these components are likely to be spatially separated (see however Lovelace and Chou 1996). In this work we assume that any co-rotating gas has been swept-out from the spatial regions (radii) occupied by counter-rotating gas. The interaction between co- and counter-rotating stars by dynamical friction is negligible because of the small densities and large relative velocities. Also, the direct interaction of co-rotating stars with counter-rotating gas is negligible because of the small cross-section of a star. Accretion occurs in the counter-rotating galaxy model of Thakar and Ryden (1996) due to viscosity of the gas which is treated using sticky-particle N-body simulations. However, the viscosity used is not derived from physical principles. Here, we analyze the influence of the two-stream instability in generating spiral waves in counter-steaming flat galaxies and suggest that these waves may give rise to an effective viscosity for counter-rotating gas.

Theoretical interest in galaxies with both co- and counter-rotating stars was stimulated by the early stability analysis of Kalnajs (1977) who found that the counter-rotating stars have a stabilizing influence on the bar forming ($m = 2$) mode in models without a massive halo. Studies of counter-streaming stability of idealized self-gravitating systems were made by a number of authors, by Bisnovatyi-Kogan et al. (1969) and Bisnovatyi-Kogan (1973) for counter-rotating stars in a finite radius cylinder, by Araki (1987) for equal populations of counter-rotating stars in finite-radius rigidly rotating Kalnajs disks, and by Lynden-Bell (1967) and Araki (1987) for a homogeneous system of counter-streaming stars following Jeans' neglect of the background potential. Perturbations of the form $f(r)\exp(im\phi - i\omega t)$ (in cylindrical coordinates (r, ϕ, z) with $m = 0, 1, ..$) were found to be linearly unstable for some conditions for the cylinder for $m = 2$, whereas for the Kalnajs disks the $m = 1$ mode appears as the main two-stream instability. For homogeneous counter-streaming systems with Maxwellian distribution functions, Lynden-Bell and Araki found that the Jeans instability always dominates the two-stream instability. N-body computer simulations of stellar disks by Zang and Hohl (1978), Sellwood and Merrit (1994), and Howard et al. (1996) support the idea of Kalnajs (1977) that counter-rotating stars have a stabilizing influence on the bar forming mode. However, the counter-

streaming is found to enhance the growth of the one-armed ($m = 1$) mode.

Section 2 of this work discuss the basic equations for the small-amplitude, tightly-wrapped spiral waves in a flat galaxy. Section 3 treats the stability of the spiral waves for different cases of a disk with co-rotating stars and counter-rotating gas and/or stars. Section 4 shows that the two-stream instability found in Sec. 3 arises from the interaction of positive and negative energy modes associated with the co- and counter-rotating matter. Section 4 discusses possible non-linear effects of the instabilities including gas accretion. Section 5 gives conclusions of this work.

2. THEORY OF SPIRAL WAVES

We first give a brief summary of the linear WKB theory of tightly-wrapped spiral waves in a single component galaxy of stars rotating with angular rate $\Omega(r) > 0$ (see for example Binney & Tremaine 1987; hereafter denoted BT; or Palmer 1994). We use an inertial, cylindrical (r, ϕ, z) coordinate system and assume a thin disk galaxy as indicated in Figure 1. In the midplane of the galaxy the perturbation of the gravitational potential is

$$\delta\Phi(r, \phi, z = 0, t) = C \ \exp\Big[\, i \int^r dr' \ k_r(r') + im\phi - i\omega t \Big], \tag{1}$$

where the radial wavenumber k_r satisfies $|k_r r| \gg 1$ for a tightly wrapped wave, $C(r)$ a slowly varying function of r, $m = \pm 1, \pm 2, ..$ is the number of spiral arms, and ω is the angular frequency of the wave. Only $m > 0$ need be considered because $\delta\Phi^*$ is a valid solution if $\delta\Phi$ is. Then, $k_r > 0$ (< 0) corresponds to a trailing (leading) spiral wave. The thin disk assumption requires $|k_r h| \ll 1$, where h is disk half-thickness.

A well-known calculation (see BT or Palmer 1994) of the linear perturbation of the dynamical equations leads to the dispersion relation

$$0 = \epsilon(\omega, k_r) \equiv 1 + \mathcal{P}_*(\omega, k_r) \ , \tag{2}$$

where

$$\mathcal{P}_* = \frac{2|\overline{k}_r| \exp(-X_*)}{X_*} \sum_{l=1,2,..} \frac{I_l(X_*)}{(s/l)^2 - 1} \ ,$$

$$s \equiv (\omega - m\Omega)/\kappa \ , \qquad \kappa^2 \equiv \frac{1}{r^3}\frac{d}{dr}(r^4\Omega^2) \ ,$$

$$\overline{k}_r \equiv k_r/k_{crit} \ , \qquad k_{crit} \equiv \frac{\kappa^2}{2\pi G \Sigma_{tot}} \ ,$$

$$X_* \equiv \left(\frac{k_r \sigma_r}{\kappa}\right)^2 = 0.28568 (Q_* \overline{k}_r)^2 \ ,$$

$$Q_* \equiv \frac{\kappa \sigma_r}{3.3583 G \Sigma_{tot}} \ .$$

Here, $\epsilon(\omega, k_r)$ has the role of a dielectric function for the disk, and $\mathcal{P}_*$ the polarization function (the $*$-subscript denotes stars); s is a dimensionless frequency; $\kappa(r)$ is the radial epicyclic frequency of a star; k_{crit} is a characteristic wavenumber, and $\overline{k}_r$ is the dimensionless radial wavenumber; $\Sigma_{tot}(r)$ is the total surface mass density of the disk; $Q_*(r)$ is Toomre's stability parameter; $\sigma_r(r)$ is the radial dispersion of the star velocities (for a stellar distribution function $f_* \propto \exp[-v_r^2/(2\sigma_r^2)]$); and I_l is the usual modified Bessel function of order l. The condition for axisymmetric ($m = 0$) stability is $Q_* > 1$ (Toomre 1964). The influence of halo matter enters through $\Omega(r)$.

Figure 2 shows wavenumber (k_r) - frequency (s) plots for two values of Q_* obtained from equation (2). The corresponding plot in BT (Figure 6-14a) and in Palmer (1994, Figure 12.2) is *incomplete* because the higher order branches, labeled $l = 2, 3, ..$, have been omitted. Further, the statement in BT that "a stellar disk has no pressure forces and therefore cannot support waves with $|s| > 1$" (page 367) is incorrect. The branches $l = 2, 3, ..$ are the analogues of the well-known Bernstein modes which propagate across a uniform magnetic field in a collisionless plasma (see for example Krall & Trivelpiece 1973). Here, the coriolis force is the analogue of the Lorentz force in a magnetized plasma. Note in particular that the Linblad resonances correspond to $s = \pm l$ or $\Omega_p = \Omega \pm l\kappa/m$, where $\Omega_p \equiv \omega/m$ is the pattern angular speed, $m = 1, 2, ..,$ *and* $l = 1, 2, ...$

For later use, we also give the dispersion relation for a single component gaseous disk with rotation rate $\Omega(r) > 0$,

$$0 = \epsilon(\omega, k_r) \equiv 1 + \mathcal{P}_g(\omega, k_r) \ , \tag{3}$$

where

$$\mathcal{P}_g = \frac{|\overline{k}_r|}{s^2 - 1 - (k_r c_s/\kappa)^2} \ ,$$

where c_s is the sound speed in the gas, and where the other quantities are the same as in equation (2). Equation (3) gives the explicit dependence $s^2 = 1 - |\overline{k}_r| + (k_r c_s/\kappa)^2$. Here, the Toomre (1964) condition for axisymmetric stability is $c_s > \kappa/(2k_{crit})$ or $Q_g \equiv \kappa c_s/(\pi G \Sigma_{tot}) > 1$.

Note that Ω^2, κ^2, and $\kappa \equiv |\kappa|$ are the same for both co- and counter-rotating disk components. Note also that Σ_{tot} is the total - gas plus stellar - mass density in all components. In that $\mathcal{P}_*$ and $\mathcal{P}_g$ are even functions of k_r, we have $\omega(k_r) = \omega(-k_r)$.

3. TWO-STREAM INSTABILITY

In the following subsections we consider the two-stream instability in a number of different limits. Specifically, we consider co-rotating stars $(+\Omega)$ and in subsection

3.1 counter-rotating gas $(-\Omega)$,

3.2 a small mass-fraction of counter-rotating gas,

3.3 a large mass-fraction of counter-rotating gas,

3.4 a small mass-fraction of counter-rotating stars,

3.5 a large mass-fraction of counter-rotating stars, and

3.6 arbitrary fractions of counter-rotating gas and stars.

3.1. Co-Rotating Stars/Counter-Rotating Gas

The dispersion relation for a two component galaxy consisting of co-rotating stars $(+\Omega)$ and counter-rotating gas $(-\Omega)$ can be written down immediately using equations (2) and (3),

$$0 = \epsilon(\omega, k_r) \equiv 1 + (1-\xi_g)\mathcal{P}_*(s, k_r) + \xi_g \mathcal{P}_g(s+w, k_r) \ , \tag{4}$$

where $w \equiv 2m\Omega/\kappa$, and where

$$\xi_g \equiv \Sigma_g/\Sigma_{tot}$$

is the fraction of the disk surface mass density in gas. To understand equation (4) it is useful to introduce the notion of *stellar modes* which obey

$$0 = 1 + (1-\xi_g)\mathcal{P}_*(s, k_r) \ , \tag{5a}$$

and *gas modes* which obey

$$0 = 1 + \xi_g \mathcal{P}_g(s+w, k_r) \ . \tag{5b}$$

Equation (5a) gives a family of curves $s = s_*(k_r)$ for the star modes, while (5b) gives curves $s = s_g(k_r)$ for the gas modes. This is shown in Figure 3 for $m = 2$.

The approximation of equations (5) breaks down near the isolated points in the (k_r, s) plane where the star and gas modes cross, $s_*(k_r) = s_g(k_r)$. At these points there is a strong resonant interaction between the modes.

3.2. Co-Rotating Stars/Low-Mass Counter-Rotating Gas

For $\xi_g \ll 1$, an approximate solution of the dispersion relation (4) near a mode-crossing point can easily be obtained as follows. To zeroth order in ξ_g, equation (4) is satisfied by (k_r, s_o) obeying $0 = 1 + \mathcal{P}_*(s_o, k_r)$. For small ξ_g, the gas response is large if $\varepsilon \equiv [(s_o + w)^2 - 1 - (k_r c_s/\kappa)^2]/[2(s_o + w)]$ is small. To first order in ξ_g, the solution of equation (4) can thus be written as $s = s_o + \delta s$ with $|\delta s| \ll |s_o|$. Taylor expanding about s_o gives

$$\delta s = \frac{\delta\omega}{\kappa} = -\varepsilon \pm \left(\varepsilon^2 + \delta s_o^2\right)^{\frac{1}{2}}, \tag{6}$$

where

$$\delta s_o^2 \equiv \frac{-\ \xi_g\ |\overline{k}_r|}{2(s_o + w)(\partial\mathcal{P}_*/\partial s_o)} \ .$$

Instability, $\omega_i \equiv Im(\omega) > 0$, occurs if $\delta s_o^2 < 0$ and $\varepsilon^2 < |\delta s_o^2|$. (The damping roots with $\omega_i < 0$ are ignored here and subsequently.) From equation (2), $\partial\mathcal{P}_*/\partial s_o = -s_o|..|$. Thus for instability $s_o(s_o + w) < 0$. Equivalently, there is instability if the frequency of the mode crossing, $\omega_o = \kappa s_o + m\Omega$, obeys

$$-\Omega < \ \frac{\omega_o}{m}\ < \Omega \ , \tag{7}$$

where ω_o/m is the pattern velocity of the perturbation. Later, in Sec. 4, we give a more general treatment of instability in counter rotating galaxies which also leads to equation (7). From equation (6), note that the maximum growth rate, $max(\omega_i) = \kappa|\delta s_o|$, occurs for $\varepsilon = 0$ and scales as $\xi_g^{\frac{1}{2}}$.

Figure 4 shows the behavior of the mode crossings for an unstable and a stable crossing of Figure 3 obtained from equation (4). For $\xi_g \ll 1$ and $c_s/\sigma_r = 1$, there are no unstable crossings for $m = 1$, while for $m = 3$ there are unstable crossings near $\overline{k}_r \approx 1.04$, 2.9, and 4.24. In all cases the unstable crossings obey equation (7).

Consider the consequence of the wave growth. For this it is useful to examine the evolution of a wave packet. The centroid of the packet is at r and at k_r in wavenumber space. The dispersion relation (4) gives $\omega_r = \omega_r(r, k_r) =$ const. which has the role of a Hamiltonian for the packet. The influence of the imaginary part of ω is discussed later. The Hamiltonian equations are

$$\frac{dr}{dt} = v_g = \frac{\partial\omega_r}{\partial k_r} \ , \qquad \frac{dk_r}{dt} = -\ \frac{\partial\omega_r}{\partial r} \ , \tag{8}$$

where v_g is the group velocity (see for example Bekefi 1966). In general, the r dependence of ω_r results from the $r-$dependence of all of the quantities, $\Omega,\ \kappa,\ k_{crit}$, etc., which enter in equation (4). To make the discussion tractable we assume that $\Omega/\kappa,\ Q_*,\ c_s/\sigma_r$, and ξ_g are independent of r. Then we have $\omega_r = \Omega(r) f(\overline{k}_r)$, where $\overline{k}_r \equiv k_r/k_{crit}(r)$ and where f is a dimensionless function. Notice that Figure 4 gives $f = \omega_r/\Omega$ as a function of $\overline{k}_r$. From equations (8), we find

$$\frac{dr}{dt} = \frac{\Omega}{k_{crit}}\ \frac{\partial f}{\partial \overline{k}_r}\ , \qquad \frac{d\overline{k}_r}{dt} = -\ \frac{\omega_r}{k_{crit}\Omega}\ \frac{\partial \Omega}{\partial r}\ . \quad (9)$$

For a flat rotation curve galaxy, $\Omega = \Omega_o(r_o/r)$ with $\Omega_o, r_o =$ constants, we find $d\overline{k}_r/dt = \omega_r/(k_{crit}r)$, and $r = r_o(\Omega_o/\omega_r)f(\overline{k}_r)$. We may take r_o to be the initial radius of the wave packet. Therefore, for the case of Figure 4a, we see that the packet moves radially outward in the unstable range of $\overline{k}_r$ and that $\overline{k}_r$ increases monotonically in that $\omega_r > 0$. It is a trailing spiral wave. (For $\overline{k}_r < 0$, $f(\overline{k}_r)$, which is an even function of $\overline{k}_r$, decreases with $\overline{k}_r$ in the corresponding unstable region and the packet moves inward while $\overline{k}_r$ increases. The wave in this case is a leading spiral.)

The maximum amplification of a wave packet results when $\overline{k}_r$ increases through all of the unstable range of $\overline{k}_r$ in which $f(\overline{k}_r) > 0$ (or < 0). This range is denoted $\Delta\overline{k}_r$. This gives an amplification factor

$$A \equiv \exp\Big(\int dt\ \omega_i \Big) = \exp\left(\frac{\Omega_o r_o}{\omega_r} \int_{\Delta\overline{k}_r} d\overline{k}_r\ k_{crit} \frac{\omega_i(\overline{k}_r)}{\Omega} \right) . \quad (10)$$

In general, A will be less than $A_{max} = \exp[2\pi(T_{gal}/T_{rot})(\omega_i/\Omega)_{max}]$, where T_{gal} is the age of the galaxy, T_{rot} is the rotation period at the considered location (r), and $(\omega_i/\Omega)_{max}$ is the maximum growth rate (as a function of $\overline{k}_r$).

For the case $m = 2$ of Figure 4a, the argument of the exponential is about $0.01(k_{crit}r_o)$. For values of k_{crit} of the order of that estimated for our galaxy $\approx 2\pi/10kpc$ at $r = 8.5kpc$ (BT), we conclude that the wave growth is insignificant. Basically, the perturbation convects out of the unstable $\overline{k}_r$ range before there is appreciable growth. The same conclusion applies to the unstable $m = 3$ mode crossings for $\xi_g \ll 1$ and $c_s/\sigma_r = 1$.

3.3. Co-Rotating Stars/High-Mass Counter-Rotating Gas

If the mass fraction of gas ξ_g is not small compared with unity, the axisymmetric stability of the disk is affected by both the sound speed in the gas (c_s) and the radial velocity spread of the stars (σ_r or Q_*). For $m = 0$, equation (4) is a function of ω^2, and the stability/instability boundary $\omega^2 = 0$ gives

$$0 = 1 - \frac{(1-\xi_g)|\overline{k}_r|}{X_*}\Big(1 - \exp(-X_*)\Big) I_o(X_*)\Big) - \frac{\xi_g|\overline{k}_r|}{1 + (k_r c_s/\kappa)^2}\ , \quad (11)$$

where $X_* = (k_r\sigma_r/\kappa)^2$ (Toomre 1964). For a given σ_r and ξ_g, the absence of solutions of equation (9) for k_r for a critical, sufficiently large c_s implies axisymmetric stability. This is shown in Figure 5. In contrast with the propagating or *convective* instability discussed above, the $m = 0$ instability gives $\omega_i > 0$ with $\omega_r = 0$ and therefore is a non-propagating or *absolute* instability (see for example Lifshitz and Pitaevskii 1981). We assume axisymmetric stability. (If this is not the case, instability appears in the $m \geq 1$ waves giving growth independent of the counter-streaming.)

Figure 6a shows the one-armed ($m = 1$) star and gas mode lines for a high mass fractions of gas and $c_s/\sigma_r = 0.316$. For the near-crossing shown in Figure 6a, the mode lines 'attract' in the vicinity of the circle to give instability as shown in Figures 6b and 6c. (Conversely, the mode lines 'repel' in the case where the crossing would be stable.) Figure 7 shows the two-armed ($m = 2$) mode lines and instability also for a high mass fraction of gas. Figure 8 shows the dependence of the maximum growth rate of the one-armed mode on ξ_g, Q_*, and c_s/σ_r.

Consider the consequence of the wave growth for the cases shown in Figures 6 ($m = 1$) and 7 ($m = 2$).. We comment first on the case of Figure 7 which is similar to the case discussed in the previous sub-section. Because $\omega_r < 0$, $f(\overline{k}_r) < 0$ and $d\overline{k}_r/dt < 0$ over the entire unstable range of $\overline{k}_r$. Over most of this range $dr/dt > 0$ for $\overline{k}_r > 0$, whereas $dr/dt < 0$ for $\overline{k}_r < 0$. Thus an unstable trailing spiral wave moves radially outward, whereas an unstable leading spiral wave moves inward. The maximum amplification from equation (10) is about $\exp[2.4(k_{crit}r_o)]$ for the trailing waves and $\exp[1.7(k_{crit}r_o)]$ for the leading waves. Thus, for $k_{crit} = 2\pi/10kpc$ and $r_o = 10kpc$, the amplification factor for the trailing waves is about 4.2×10^6.

The case of Figure 6b for $m = 1$ and $\xi_g = 0.25$ is different because $\omega(\overline{k}_r)$ changes sign at $\overline{k}_0$ within

the unstable range of $\overline{k}_r$. Recall that for a flat rotation curve, $r = r_o(\Omega/\omega_r)f(\overline{k}_r)$, where r_o is the initial radius of the wave packet. Thus, for $\overline{k}_r < \overline{k}_0$ in the unstable range and $\overline{k}_r > 0$, we must have $\omega_r = const. > 0$, so that $f(\overline{k}_r) > 0$ and $d\overline{k}_r/dt > 0$. However, as $\overline{k}_r$ increases, $f(\overline{k}_r)$ eventually decreases and approaches zero so that $r \to 0$; that is, the wave packet moves towards the center of the galaxy. On the other hand, for $\overline{k}_r > \overline{k}_0$ in the unstable range, we must have $\omega_r = const. < 0$ so that $f(\overline{k}_r) < 0$, and $d\overline{k}_r/dt < 0$. In this limit, $\overline{k}_r$ decreases and $f(\overline{k}_r)$ eventually increases and approaches zero at $\overline{k}_0$ so that again $r \to 0$. For both $\overline{k}_r > \overline{k}_0$ and $< \overline{k}_0$, the wave amplification is even larger than than that for the above-mentioned $m = 2$ instability for the same ξ_g. Thus, it is likely that $A = A_{max}$. The behavior of the trailing waves, $\overline{k}_r < 0$, is different. For both $\overline{k}_r < -\overline{k}_0$ and $> -\overline{k}_0$, $\overline{k}_r$ moves away from $-\overline{k}_0$ and the wave packet moves from its initial radius to larger distances. The wave amplification from equation (10) in this case is even larger than that for the trailing waves because $|\Omega_o/\omega_r| \gg 1$ in equation (10). When the wave-packet reaches and passes the right or left-hand limit of the unstable range of $\overline{k}_r$ for $\overline{k}_r < 0$, the packet continues to propagate adiabatically (without growth) away from $\overline{k}_0$ with part of the wave energy in the upper frequency (ω_r) branch and part in the lower branch. For example, $\overline{k}_r$ larger than the right-hand limit the wave on the lower frequency branch moves towards the center of the galaxy whereas the wave on the upper branch moves to larger radii.

For the case of Figure 6c for $m = 1$ and $\xi_g = 0.15$, ω_r is positive throughout the unstable range of $\overline{k}_r$. The maximum amplification factor for the trailing waves ($\overline{k}_r > 0$) is about $\exp[3.5(k_{crit}r_o)]$, whereas for the leading waves ($\overline{k}_r < 0$) it is about $\exp[8.67(k_{crit}r_o)]$. Both factors are very large for $k_{crit}r_o \sim 2\pi$, but that for the leading waves is strongly dominant.

Consider the $m = 1$ wave growth for large mass-fractions of gas, say, $\xi_g = 0.3 - 0.7$. Stability of the $m = 0$ Toomre mode requires larger values of Q_* and/or c_s/σ_r than considered above (see Fig. 5), and here we consider $Q_* = 1.8$ and $c_s/\sigma_r = 0.5$. For $\xi_g = 0.3$, the unstable waves occur for $\overline{k}_r = 0.72 - 1.46$, the maximum growth rate is $\omega_i/\Omega = 0.17$, ω_r/Ω varies from 0.092 to 0.096. For $\xi_g = 0.5$, the unstable range is $\overline{k}_r = 0.63 - 2.0$, the maximum growth rate is $\omega_i/\Omega = 0.265$, and ω_r/Ω varies from zero to -0.042. For $\xi_g = 0.7$, the unstable range is $\overline{k}_r = 0.64 - 2.56$, the maximum growth rate is $\omega_i/\Omega = 0.29$, and ω_r/Ω varies from -0.13 to -0.2. Although the growth rate is somewhat larger for $\xi_g = 0.7$, the wave amplification is the maximum possible (A_{max}) for the $\xi_g = 0.5$ case where the wave frequency $\omega_r(\overline{k}_r)$ goes to zero in the unstable range of $\overline{k}_r$ (see equation 10).

In Section 5 we discuss non-linear processes which may lead to saturation of the wave growth.

3.4. Co-Rotating Stars/Low-Mass Counter-Rotating Stars

The dispersion relation for a two component galaxy consisting of co-rotating ($+\Omega$) stars and a mass fraction $\xi_* \leq \frac{1}{2}$ of counter-rotating stars ($-\Omega$) is

$$0 = \epsilon(\omega, k_r) \equiv 1 + (1-\xi_*)\mathcal{P}_*(s, k_r) + \xi_*\mathcal{P}_*(s+w, k_r) \ , \tag{12}$$

where $w \equiv 2m\Omega/\kappa$ and $\mathcal{P}_*$ is defined below equation (2). The co-rotating star modes are given by $0 = 1 + (1-\xi_*)\mathcal{P}_*(s, k_r)$, while the counter-rotating star modes are given by $0 = 1 + \xi_*\mathcal{P}_*(s+w, k_r)$.

Figure 9a shows the behavior of the two-armed ($m = 2$) modes for $\xi_* \ll 1$ and $Q_* = 1.4$ for both components. The circled mode crossings in this figure satisfy equation (7) suggesting instability. The instability is confirmed by Figure 9b which shows the complex ω as a function of $\overline{k}_r$ obtained from equation (12) for $\xi_* = 0.05$ and $Q_* = 1.4$ for both components. From equation (10), the largest amplification factor is about $\exp[0.18(k_{crit}r_o)]$ for the wave with $\omega_r < 0$. For $k_{crit}r_o \sim 2\pi$, this amplification is insignificant. For the same conditions, the three-armed spiral waves are stable. The one-armed waves are unstable for $\overline{k}_r = 0.935$ to 1.5, with a maximum growth rate of $\omega_i/\Omega = 0.079$ at $\overline{k}_r = 1.25$ where $\omega_r/\Omega = 0.27$, and with maximum amplification factor $\exp[0.13(k_{crit}r_o)]$, which is insignificant for $k_{crit}r_o \sim 2\pi$.

3.5. Comparable Mass Co-/Counter-Rotating Stars

Figure 10a shows the behavior of the two-armed ($m = 2$) modes for $\xi_* = \frac{1}{2}$ and $Q_* = 1.4$ for both components. Again the circled mode crossings in this figure satisfy equation (7) suggesting instability. The instability is confirmed by Figure 10b which shows ω_r and ω_i as a function of $\overline{k}_r$ obtained from equation (12). From equation (10), the maximum amplification factor is about $\exp[0.58(k_{crit}r_o)]$. Note that for $\xi_* = \frac{1}{2}$, the waves are completely symmetric under $\phi \to -\phi$: the co-rotating wave ($\omega_+ = \omega_r + i\omega_i$, $\omega_r >$

0) is matched by an equivalent counter-rotating wave ($\omega_- = -\omega_r + i\omega_i$) with the same growth rate. Figure 11 shows the dependence of the maximum growth rate on Q_*.

The three-armed waves are stable for $\xi_* = \frac{1}{2}$ and $Q_* = 1.4$ for both components. However, the one-armed waves are strongly unstable for the same conditions. The mode lines shown in Figure 12a do not cross but 'attract' (as noted earlier) near the circled point to give instability as shown in Figure 12b. The exact symmetry $\phi \to -\phi$ for $\xi_* = \frac{1}{2}$ makes it evident that the co- and counter-rotating mode lines in Figure 12a must merge at $s = -\Omega/\kappa$ to give $\omega_r = 0$ (independent of Ω/κ) in the range of $\overline{k}_r$ where $\omega_i > 0$. Thus the $m = 1$ instability in this case is an *absolute* instability. In this respect it is similar to the $m = 0$ instability. Trailing and leading wave perturbations can be superposed to give a standing wave in place of equation (1), $\delta\Phi = C\cos(\int^r dr' k_r(r'))\sin(\phi)\exp(\omega_i t)$. Figure 13 shows the dependence of the maximum growth rate on Q_* for $\xi_* = \frac{1}{2}$.

For $\xi_* = \frac{1}{2}$, but unequal velocity spreads of the components, the symmetry $\phi \to -\phi$ is spoiled. For example, the counter-rotating stars may be younger with a smaller velocity spread. For $\sigma_r^- = 0.316\sigma_r^+$ ($Q_*^- = 0.316 Q_*^+$), we find that stability of the $m = 0$ Toomre mode requires $Q_*^+ > 1.86$, while the $m = 1$ mode is unstable for $Q_*^+ < 2.85$. At ¿$Q_*^+ = 1.86$, the $m = 1$ mode has $\omega_i/\Omega \approx 0.34$ and $\omega_r/\Omega \approx = -0.2$. In this case the $m = 1$ instability is *convective.*

For $\xi_* < \frac{1}{2}$, the one-armed waves still have large growth rates if say $\xi_* > 0.05$. This is shown in Figure 14a. However, ω_r takes on positive values due to the stronger 'pull' of the co-rotating stars. With $\omega_r > 0$, $\overline{k}_r$ increases through the unstable range (see equation 9) for both trailing ($\overline{k}_r > 0$) and leading ($\overline{k}_r < 0$) waves. Thus the instability is *convective.* There is a weak dependence of ω_r/Ω on $\overline{k}_r$ which is such that the centroid of a wave packet moves outward and then inward over a small range of r as $\overline{k}_r$ increases through the unstable range. The maximum amplification factor from equation (10) is shown in Figure 14b.

For $\xi_* > \frac{1}{2}$, the solutions are given by those for $\xi_* < \frac{1}{2}$ by the replacements $\xi_* \to 1 - \xi_*$ and $(\omega_r,\ \omega_i) \to (-\omega_r,\ \omega_i)$, provided that the velocity dispersions (the $Q_*'s$) of the two components are the same.

3.6. Co-Rotating Stars/Counter-Rotating Gas and Stars

The dispersion relation for a three component galaxy consisting of co-rotating stars and a mass fraction ξ_g of counter-rotating gas and a fraction ξ_* of counter-rotating stars is

$$0 = \epsilon(\omega, k_r) \equiv 1 + (1-\xi_g-\xi_*)\mathcal{P}_*(s, k_r) + \xi_g \mathcal{P}_g(s+w, k_r) + \xi_* \mathcal{P}_*(s+w, k_r)\ , \qquad (12)$$

where $w \equiv 2m\Omega/\kappa$ and $\mathcal{P}_*$ is defined below equation (2). The co-rotating star modes are given by $0 \approx 1 + (1-\xi_g-\xi_*)\mathcal{P}_*(s, k_r)$, while the counter-rotating gas modes are given by $0 = 1 + \xi_g\mathcal{P}_g(s+w, k_r)$ and the counter-rotating star modes are given by $0 = 1 + \xi_*\mathcal{P}_*(s+w, k_r)$.

Figure 15 shows the real and imaginary parts of ω as a function of $\overline{k}_r$ for one-armed waves for a sample case where $\xi_g = \xi_* = 0.1$, with $Q_* = 1.4$ for both star components, and $c_s/\sigma_r = 0.316$ for the gas. The instability arises mainly from the interaction of the co-rotating stars and the counter-rotating gas due to the lower velocity dispersion of the gas. The maximum amplification factor for the trailing waves ($\overline{k}_r > 0$) is about $\exp[1.23(k_{crit}r_o)]$, whereas for the leading waves ($\overline{k}_r < 0$) it is about $\exp[1.7(k_{crit}r_o)]$. For $k_{crit}r_o = 2\pi$, these factors are 2.3×10^3 and 4.3×10^4, respectively.

Figure 16 shows the dependence of the one-armed ($m = 1$) wave instability on ξ_g for three values of ξ_*. In this figure, $Q_* = 1.6$ for both stellar components, and $c_s/\sigma_r = 0.5$. From the discussion of Sec. 3.3, it is clear that the largest amplification occurs when $\omega_r = 0$ within the unstable $\overline{k}_r$ range. Therefore, it follows from this Figure 16b that the largest amplification occurs for $\xi_g + \xi_* \approx \frac{1}{2}$. Under this condition the amplification is largest for the leading waves ($\overline{k}_r < 0$). The growth rates and amplification factors are smaller for the $m = 2$ waves.

4. POSITIVE/NEGATIVE MODE ENERGY OF UNSTABLE WAVES

The energy-density of an electrostatic wave in a homogeneous plasma is given by the well-known expression $\mathcal{E}_w = \omega_r(\partial\epsilon/\partial\omega_r)|\delta\mathbf{E}|^2/(8\pi)$, where ϵ is the dielectric function, $\delta\mathbf{E} = -\nabla\delta\Phi_e$, and $\nabla^2\delta\Phi_e = -4\pi\delta\rho_e$ (Coppi, Rosenbluth, & Sudan 1969). The corresponding expression for a self-gravitating medium differs by a minus sign and is $\mathcal{E}_w = -\omega_r(\partial\epsilon/\partial\omega_r)|\delta\mathbf{g}|^2/$

$(8\pi G)$, where $\delta\mathbf{g} = -\nabla\delta\Phi$ and $\nabla^2\delta\Phi = +4\pi G\delta\rho$. The generalization to tightly wrapped waves in a flat galaxy gives the energy (per unit area of the disk)

$$\mathcal{E}_w = -\omega_r(\partial\epsilon/\partial\omega_r)\pi G|\delta\Sigma|^2/|k_r| \ , \tag{13}$$

where we have used $\delta\Phi = -2\pi G\delta\Sigma/|k_r|$, and $\delta\Sigma$ is the total surface mass-density perturbation. An equation equivalent to (13) is derived by Shu (1992).

For a galaxy consisting of co- and counter-rotating components (now denoted for generality by + and −), we have $\epsilon = 1 + \mathcal{P}^+ + \mathcal{P}^-$. Thus, the wave energy-density is the sum of the energy densities associated with the modes in the co- and counter-rotating components,

$$\mathcal{E}_w = \mathcal{E}_w^+ + \mathcal{E}_w^- ,$$
$$\mathcal{E}_w^\pm = -\omega_r(\partial\mathcal{P}^\pm/\partial\omega_r)\pi G|\delta\Sigma|^2/|k_r| \ . \tag{14}$$

If the signs of $\mathcal{E}_w^+$ and $\mathcal{E}_w^-$ are different, we have the necessary condition for the well-known instability of interacting positive and negative energy modes (Coppi et al. 1969). The negative energy mode can grow by feeding energy into the positive energy mode. A plasma example is the two-stream instability which can occur when a beam of charged particles passes with velocity v_b through a background plasma. In this case the energies associated with the modes in the beam and the background plasma have opposite signs. The mechanism of this instability can be understood by considering a localized excess or clump of say positive charge in the beam which will induce a negative clump in the background. The relative motion of the clumps leads to an electrostatic attraction between them, a slowing down of the beam clump, a speeding up of the background clump, and a growth of the electric field energy. The clump size must be sufficiently large to give instability, larger than about $\pi v_b/\omega_p$ for the case of equal beam and plasma densities, where ω_p is the plasma frequency. In a *homogeneous* self-gravitating system involving a beam and a background, a density excess in the beam will induce an excess in the background, and it would appear that a similar two-stream instability would occur. However, for most distribution functions, the large clump size needed for the two-stream instability ($> \pi v_b/\omega_J$, where $\omega_J = (4\pi G\rho)^{\frac{1}{2}}$ is the Jeans angular frequency) results instead in the Jeans instability being dominant (Lynden-Bell 1967, Araki 1987). The Jeans instability occurs for clump sizes larger than about $\pi\Delta v/\omega_J$, where Δv is the velocity spread. In contrast, in a disk galaxy the velocity spreads of the $\pm\Omega$ components *and* the disk rotation $[(\kappa^+)^2 = (\kappa^-)^2 > 0]$ provide stability to the Jeans instability ($m = 0$) (Toomre 1964).

From equations (2) and (3), the condition for $\mathcal{E}_w^+\mathcal{E}_w^- < 0$ is simply $s_r(s_r + w) < 0$ or $-\Omega < \omega_r/m < \Omega$ which is equation (7). Recall that ω_r/m is the pattern or angular phase velocity of the wave. For the two-stream instability in a beam-plasma system, the phase velocity of the unstable waves is also intermediate between velocity of the beam and that of the background.

The axial angular momentum of the wave (per unit area of the disk) can be written as $\mathcal{J}_w = \mathcal{J}_w^+ + \mathcal{J}_w^-$, where $\mathcal{J}_w^\pm = -m(\partial\mathcal{P}^\pm/\partial\omega_r)\pi G|\delta\Sigma|^2/|k_r|$ (Coppi et al. 1969), so that $\mathcal{E}_w = (\omega_r/m)\mathcal{J}_w$. Thus, the wave angular momenta associated with the modes in the co- and counter- rotating components have opposite signs under the same condition that the mode energies have opposite signs. Under this condition, the angular momentum of the mode in the co-rotating component is negative, whereas that in the counter-rotating component is positive. Thus, the total angular momentum of the mode plus matter of the $+\Omega$ component is reduced, whereas that in the $-\Omega$ component is increased (but decreased in magnitude). The situation is analogous for the beam-plasma instability where the linear momentum of the beam mode is negative and that of the background mode is positive.

5. NON-LINEAR EFFECTS AND GAS ACCRETION

The non-axisymmetric instabilities discussed here are of interest because they could be important for the inward radial transport or accretion of counter-rotating $(-\Omega)$ matter deposited or accreted at large r onto an existing flat galaxy consisting mainly of co-rotating $(+\Omega)$ stars. We consider first the case where the counter-rotating matter is gas.

The linear wave growth is determined mainly by three parameters, the Toomre parameter Q_* for the stars, the ratio of the sound speed in the gas to the radial velocity spread of the stars c_s/σ_r, and the mass fraction of the counter-rotating gas ξ_g. In general these quantities will vary with r; the theory is still valid as long as the variation is small on a scale $1/k_r$. For a given ξ_g, the wave growth rate decreases monotonically with increasing Q_* and c_s/σ_r, as shown for example by Figure 8 for one-armed waves. On the other hand, with Q_* and c_s/σ_r fixed, the wave growth is zero for ξ_g below a threshold value and increases

strongly as ξ_g increases above this value as shown by Figure 8a.

Suppose that Q_*, c_s/σ_r, and ξ_g are such as to give a large growth rate, say, $\omega_i/\Omega > 0.05$, and an appreciable wave amplification factor, $A \gg 1$. After several rotation periods of the matter at a radius r, the wave amplitude can grow to a level where non-linear effects become important ($|\delta\Phi| \sim (0.01 - 0.05)(\Omega r)^2$). One non-linear effect is the scattering of stars into less circular orbits which acts to increase the radial velocity spread and therefore Q_*. In turn, the increase of Q_* acts to reduce the growth rates. However, a finite level of non-axisymmetric waves may remain excited due to reduced but non-zero wave growth. These waves can give an effective viscosity ν_e which causes inward transport or accretion of the gas and outward transport of the gas's angular momentum. The gas can remain in approximately circular motion by radiating the energy excess which results from its inward motion. This gives an accretion luminosity (per unit area of the disk) $\Sigma_g \Omega^2 r |\overline{v}_{gr}|$, assuming a flat rotation curve, where $\overline{v}_{gr} < 0$ is the average accretion speed of the gas. A rough estimation using quasi-linear theory gives $\nu_e \sim \Omega r^2 |k_r r|^3 |\delta\Phi/(\Omega r)^2|^2$ and $\overline{v}_{gr} \sim -\nu_e/r$, where $\delta\Phi$ is the residual wave level. Relevant values for accretion from say $r = 20$ kpc in 3×10^9 yrs. are $|v_{gr}| > 5$ km/s.

A second non-linear effect is the steepening of the spiral wave in the gas which can lead to the formation of a oblique shock as indicated in Figure 17 for the case of a one-armed leading wave. (The amplification factor for the leading wave is generally larger than that for the trailing wave.) The gas velocity parallel to the shock is unchanged across the shock. Up stream of the shock, the normal component of the velocity $v_{gn} = \theta\Omega r$ is larger than the ambient sounds speed c_s^o in order to have a shock. Here, $\theta = |k_r r|^{-1} \ll 1$ is the pitch angle of the spiral arm. That is, the normal Mach number $M_n = v_{gn}/c_s^o > 1$. We assume that M_n is not much larger than unity so that the shock is not very strong. Downstream from the shock, the normal velocity of the gas is less than the post-shock sound speed. As a result, the gas acquires an inward radial velocity $v_{gr} = -\beta\Omega r$, where $\beta = \frac{2}{\gamma+1}(1 - 1/M_n^2)\theta$ and γ is the adiabatic index. This shock deflection could be important for the inward radial transport of counter-rotating gas. The angular momentum lost by the gas may in be transported outward by the spiral wave. Also in this case, the gas can remain in approximately circular motion by radiating the energy excess arising from accretion. Across the disk, the azimuthally averaged accretion speed $\overline{v}_{gr}$ has a strong non-linear dependence on ξ_g because at small ξ_g there is no instability and no transport, while for $\xi_g \approx \frac{1}{2}$ there is maximum wave amplification (Sec. 3.3). As a result, the gas transport equation $\partial\Sigma_g/\partial t + (1/r)\partial[r\Sigma_g \overline{v}_{gr}(\Sigma_g)]/\partial r = 0$ (neglecting star formation) is strongly non-linear. Its solutions may involve inward propagating soliton-like perturbations in Σ_g. This may be the cause of ring-like features observed in some counter-rotating galaxies (for example, NGC 7217, Buta et al. 1995, and NGC 4138, Jore et al. 1996).

The growth of spiral arms with significant self-gravity may be important in inducing collisions and agglomeration of gas clouds in the arms (see for example Roberts, Lowe, & Adler 1990) thereby enhancing the rate of formation of counter-rotating stars. Newly formed counter-rotating stars should have a smaller velocity spread than the older co-rotating stars which have undergone stochastic heating (BT, p. 484).

If the counter-rotating matter consists of stars, the $m = 1$ instability discussed in Sec. 3.2 may be important. From Figure 14, the instability is significant for a mass fraction of counter-rotating stars ξ_* larger than 0.05 and is strongest for $\xi_* = \frac{1}{2}$ if $Q_* = 1.4$ for both components. For ξ_* increasing from $\frac{1}{2}$, the growth rate decreases. The large growth rates and amplification factors of Figure 14 we expect to exist only transiently because the resulting waves would rapidly scatter stars into less circular orbits increasing Q_* and decreasing ω_i. However, a finite level of waves may remain excited due to much smaller growth rates. Consider now the possibility of radial inflow of the stars. A counter-rotating star at a radius r with angular momentum $l_z^- < 0$ can lose angular momentum $\delta l_z^- > 0$ to the spiral wave in a given period of time so that the magnitude of l_z^- decreases. At the same time, a co-rotating star also at r with angular momentum $l_z^+ > 0$ can lose $\delta l_z^+ < 0$ so that the magnitude of l_z^+ also decreases. The resulting smaller values of $|l_z^{\pm}|$ correspond to smaller mean radii. In order for a co- or counter-rotating star to stay in approximately circular motion, its energy must decrease by an amount $\delta e^{\pm} = \Omega \delta l_z^{\pm} < 0$, assuming a flat rotation curve. However, the ratio of the energy to the angular momentum of the spiral wave $\mathcal{E}_w/\mathcal{J}_w = \omega_r/m = \omega_r \ll \Omega$ from Sec. 4 and Figure 14. That is, the energy loss of the stars to the spiral wave is too small to allow inward radial motion on approximately circular or-

bits. Even a small fractional inward motion of the stars will increase Q_* sufficiently to stabilize the spiral wave modes.

The stability of a galaxy with both counter-rotating stars and gas (Sec. 3.6) appears qualitatively similar to the results found with only counter-rotating gas. The largest wave amplifications are for the $m = 1$ leading spiral waves and for $\xi_* + \xi_g \approx \frac{1}{2}$. The above discussion of gas transport is also pertinent to this case.

6. CONCLUSIONS

The two-stream instability - well-known in plasma physics - has been discussed by a number of authors for counter-streaming self-gravitating systems. However, a somewhat muddled picture has emerged for the role of this instability in galaxies with counter-rotating components including the notion that the instability does not occur due to the difference between the universal attraction of masses and the repulsion/attraction of like/unlike charges. Indeed, stability analysis of a homogeneous self-gravitating system of counter-streaming particles (following Jeans' neglect of the background potential), shows that the two-stream instability is dominated by the Jeans instability if the distribution functions are Maxwellian (Lynden-Bell 1967, Araki 1987). However, the homogeneous system results are *not* relevant to counter-stream matter in disk galaxies where the Jeans instability ($m = 0$) is stabilized by both the velocity spreads of the two components *and* the disk rotation ($\kappa^2 > 0$ for both components) (Toomre 1964). A study by Araki (1987) of the stability of counter-streaming, rigidly-rotating, finite-radius Kalnajs disks does in fact show an $m = 1$ two-stream instability, but the physical nature of the instability is not elucidated. Simulation studies of different galaxy models by Zang and Hohl (1978), Sellwood and Merritt (1994), and Howard et al. (1996) all show clear evidence of a strong $m = 1$ two-stream instability.

In this work, we first briefly summarize known results for small-amplitude, tightly-wrapped spiral waves in a single component stellar disk and show (in Sec. 2) that the higher order modes of the dispersion relation have been overlooked in standard treatments (BT, Palmer 1994). The dispersion relation gives the dependence of the wave frequency (ω) on the radial wavenumber (k_r), and the omitted curves are labeled $l = 2, 3, ..$ in Figure 2. These higher order modes are the analogues of the Bernstein modes in a collisionless plasma which propagate across a uniform magnetic field. Their existence leads to a richer set of Linblad resonances which occur at pattern speeds $\Omega_p \equiv \omega/m = \Omega \pm l\kappa/m$ with $m = 1, 2, ..$ *and* $l = 1, 2, ...$

We assume tightly wrapped spiral waves, $|k_r|r \gg 1$, so that the WKB approximation holds, but it is not known how strong this inequality must be. On the other hand, for effective swing amplification involving radial reflection of waves, which is thought to drive spiral waves in galaxies with only co-rotating matter, one needs $|k_r|r < 3$ (BT, p. 376). Here, the two-stream wave amplification is sufficiently large so as to be important without wave reflection. Although selection effects may be involved, observed counter-rotating disk galaxies are early-type spirals or S0's which do not show prominent open spiral structure.

We go on to discuss spiral waves in flat galaxies with counter-streaming where there is a co-rotating stellar component with angular velocity $+\Omega$ and a counter-rotating component(s) of gas and/or stars with angular velocity $-\Omega$. We then plot on the same graph the $\omega = \omega(k_r)$ lines - referred to as mode lines - for the co-rotating stellar component (neglecting the counter-component) and the $\omega = \omega(k_r)$ lines for the counter-component (neglecting the co-component). At crossing points of these mode lines in the (k_r, ω) plane, a strong resonant interaction can occur between co- and counter-rotating components. Direct solution of the dispersion relations (Sec. 3) shows that the crossings are unstable if the pattern angular speed of the spiral wave ω/m is such that $-\Omega < \omega/m < \Omega$. The 'mode-crossings' and near 'mode-crossings' (Sec. 3.3) satisfying this condition give the two-stream instability in a counter-rotating galaxy. They arise from the fact that the mode energies for the co- and counter-rotating components have opposite signs for $-\Omega < \omega/m < \Omega$ (Sec. 4). This is analogous to the two-stream instability in a beam-plasma system where the mode energies for the beam and the background have opposite signs for a wave phase velocity intermediate between that of the beam and that of the background.

Instability of a spiral wave may or may not be significant because the waves are in most cases *convective* in the sense that both the position (r) and the wavenumber (k_r) of an unstable wave packet evolve with time (Sec. 3.2). More important than the local growth rate (ω_i) is the amplification factor of the advected wave A (equation 10) which is the maxi-

mum factor by which an unstable packet can grow before being convected out of the unstable range of k_r. In most cases, we find that the amplification factors are largest for one-armed $m = 1$ leading spiral waves (with respect to the co-rotating stars). However, for larger stellar velocity spreads (larger Q_*), the two-armed $m = 2$ wave may be unstable, with the largest amplification for the trailing wave, while the $m = 1$ wave is stable. The growth rates and amplification factors increase as the mass-fraction of counter-rotating gas ξ_g or of stars ξ_* increases if these fractions are less than $\approx \frac{1}{2}$. On the other hand, both ω_i and A decrease as the velocity spreads of the stars and/or gas increase.

For the case of only counter-rotating gas of mass fraction $\xi_g <\approx \frac{1}{2}$, the largest amplification factors are for the $m = 1$ leading spiral waves (Sec. 3.3). For the case of only counter-rotating stars of mass-fraction $\xi_* <\approx \frac{1}{2}$, the largest ω_i and A values occur also for the $m = 1$ leading spiral waves (Sec. 3.5). If the velocity spreads in the two stellar components are equal, then for $\xi_* = \frac{1}{2}$, the $m = 1$ instability is an *absolute* or non-propagating instability, the real part of the wave frequency (ω_r) is zero, and the growth rate has its maximum value. For $\xi_* > \frac{1}{2}$, the instability is again convective, the amplification factors are largest for the trailing spiral waves, and ω_i and A decrease with increasing ξ_*. These results are in qualitative accord with the theoretical results of Araki (1987) for counter-rotating stellar disks and with the simulation results of Sellwood and Merritt (1994), where $\xi_* = \frac{1}{2}$ in both studies.

For a galaxy with both counter-rotating stars and gas, the largest amplification is for the $m = 1$ leading spiral waves, and it occurs when the counter-rotating mass fraction is $\xi_* + \xi_g \approx \frac{1}{2}$ (Sec. 3.6).

Possible non-linear effects which act to limit the wave growth and amplification are discussed in Sec. 5. The effects include scattering of star orbits by the wave which increases Q_*, and heating of the gas. A residual level of spiral waves may remain excited which give an effective viscosity for the gas causing its accretion. Also, a leading $m = 1$ spiral shock wave may form in the gas causing its accretion. However, from the considerations of Sec. 5, it appears unlikely that the spiral waves cause accretion of counter-rotating stars.

The gas viscosity (and thus accretion rate) due to the two-stream waves is plausibly an increasing function of the spiral wave amplification factor. With no amplification there are no waves and no viscosity. Thus, the fact that the largest amplification occurs for a mass-fraction of counter-rotating matter $\xi_* + \xi_g \approx \frac{1}{2}$ may be pertinent to the counter-rotating galaxy NGC-4550 (Rubin et al. 1992) which is remarkably symmetric between co- and counter-rotating stars, $\xi_* \approx \frac{1}{2}$ (Rix et al. 1992). A schematic scenario is that initially counter-rotating gas is supplied at large r and has a fastest accretion rate for $\xi_g = \frac{1}{2}$. At later times, after counter-rotating stars have formed with mass fraction ξ_*, the fastest accretion rate is for $\xi_g = \frac{1}{2} - \xi_*$. At even later times, the gas accretion ceases when $\xi_* \approx \frac{1}{2}$.

Accretion of gas due to two-stream waves is strongly nonlinear when ξ_g is large and this may lead to a 'pile up' of gas into rings at one or more radial distances. Prominent rings are seen in NGC 3593, NGC 4138, and NGC 7217. Buta et al.(1995) have suggested that the locations of the three rings in NGC 7217 correspond to different Linblad resonances. At sufficiently high counter-rotating gas densities counter-rotating star formation may be triggered.

We thank G. Contopoulos, H.H. Fleischmann, B.R. Kusse, W.I. Newman, J.A. Sellwood, and A.R. Thakar for valuable discussions. This work was supported in part by NSF grant AST-9320068.

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Fig. 1.— Sketch of the counter rotating disk geometry.

Fig. 2.— The figure shows the frequency [$s \equiv (\omega - m\Omega)/\kappa$] - wavenumber ($k_r$) dependence of the tightly wrapped spiral wave modes in a disk of stars. Here, Ω is the angular velocity of the stars, κ is their epicyclic frequency, k_{crit} is the critical wavenumber, and l labels different branches of the dispersion relation. The higher branches ($l \geq 2$) are analogues of the Bernstein modes in a magnetized plasma as discussed in the text.

Fig. 3.— The figure shows the different star and gas two-armed ($m = 2$) modes in a galaxy of co-rotating stars with $Q_* = 1.4$ and a small mass fraction of counter-rotating gas ($\xi_g \ll 1$). The curves extend to negative k_r as even functions. At the circled points where the star and gas modes cross there is a strong resonant interaction which may lead to instability. Here, only the point B is unstable. The detailed behavior of the mode crossings is shown in Figure 4.

Fig. 4.— The figure shows the behavior of the star/gas mode crossings for two armed waves ($m = 2$) and a small mass fraction of gas, $\xi_g = 0.01$. Also, $Q_* = 1.4$, $c_s/\sigma_r = 1$, and we have assumed a flat rotation curve so that $\kappa = \sqrt{2}\Omega$. **a** is for an unstable case (point B of Figure 3), and **b** for a stable case (point C of Figure 3).

Fig. 5.— The figure shows the dependence of the axisymmetric ($m = 0$) Toomre (1964) stability threshold on the sound speed in the gas (c_s) obtained from equation (9). Here, σ_r is the radial velocity dispersion of the stars, ξ_g is the mass fraction of the gas, and Q_* is the Toomre stability parameter for the stars defined below equation (2). We have assumed $\kappa = \sqrt{2}\Omega$. Note that the dashed curve $\xi_g = 1$ is given by $c_s/\sigma_r = 0.9355/Q_*$, which is the same as $c_s = \kappa/(2k_{crit})$ from equations (2) and (3).

Fig. 6.— The figure shows the nature of the one-armed ($m = 1$) modes in a galaxy of co-rotating stars and an appreciable mass fractions of counter- rotating gas. For this figure, $Q_* = 1.4$, $c_s/\sigma_r = 0.316$, and $\kappa = \sqrt{2}\Omega$. **a** shows the star and gas mode lines. The circle indicates the near crossing which is unstable. **b** shows the real and imaginary parts of ω obtained from equation (4) for $\xi_g = 0.25$, and **c** shows the same quantities for $\xi_g = 0.15$.

Fig. 7.— The figure shows the nature of the two-armed ($m = 2$) modes in a galaxy of co-rotating stars and an appreciable mass fraction ($\xi_g = 0.25$) of counter- rotating gas. For this figure, $Q_* = 1.4$, $c_s/\sigma_r = 0.316$, and $\kappa = \sqrt{2}\Omega$. **a** shows the star and gas mode lines. The circles indicate unstable mode crossings. **b** shows the real and imaginary parts of ω obtained from equation (4).

Fig. 8.— The figure shows the dependences of the maximum growth rate of the one-armed waves on ξ_g, Q_*, and c_s/σ_r. The left-hand limit in **b** at $c_s/\sigma_r = 0.2$ is close to the stability threshold of the axisymmetric instability (see Fig. 5).

Fig. 9.— The figure shows the nature of the two-armed ($m = 2$) modes in a galaxy of co-rotating stars ($+\Omega$) and a small fraction ($\xi_* \ll 1$) of counter-rotating stars ($-\Omega$). **a** shows the mode lines with the unstable crossings circled. **b** shows the real and imaginary parts of ω obtained from equation (12). For both star components, $Q_* = 1.4$, and we have taken $\kappa = \sqrt{2}\Omega$. The vertical arrows on the ω_i curves indicate the associated ω_r curves.

Fig. 10.— The figure shows the nature of the two-armed ($m = 2$) modes in a galaxy of co-rotating stars ($+\Omega$) and an equal mass ($\xi_* = \frac{1}{2}$) of counter-rotating stars ($-\Omega$). **a** shows the mode lines with the unstable crossing circled. **b** shows the real and imaginary parts of the wave frequency ω as a function of $\overline{k}_r$ obtained from equation (12). For both star components, $Q_* = 1.4$. Also, we have taken $\kappa = \sqrt{2}\Omega$. The vertical arrows on the ω_i curves point towards the associated ω_r curves.

Fig. 11.— The figure shows the Q_* dependence of the maximum growth rate of the two-armed waves for $\xi_* = \frac{1}{2}$. The value of k_r for the maximum growth is also indicated. We have taken $\kappa = \sqrt{2}\Omega$.

Fig. 12.— The figure shows the nature of the one-armed ($m = 2$) modes in a galaxy of co-rotating stars ($+\Omega$) and an equal mass ($\xi_* = \frac{1}{2}$) of counter-rotating stars ($-\Omega$). **a** shows the mode lines, and the circle indicates the unstable near crossing of mode lines. **b** shows the real and imaginary parts of the wave frequency ω as a function of $\overline{k}_r$ obtained from equation (12). For both star components, $Q_* = 1.4$.

Fig. 13.— The figure shows the Q_* dependence of the maximum growth rate of the one-armed waves

for a galaxy of equal mass co- and counter-rotating components, $\xi_* = \frac{1}{2}$.

Fig. 14.— The figure shows the maximum growth rate and amplification factor of the one-armed waves for a galaxy of co-rotating stars and an appreciable mass-fraction ξ_* of counter-rotating stars.

Fig. 15.— The figure shows the nature of the one armed unstable mode of galaxy of co-rotating stars and a fraction $\xi_g = 0.1$ of counter-rotating gas and a fraction $\xi_* = 0.1$ of counter-rotating stars. For this figure, $Q_* = 1.4$ for both star components, $c_s/\sigma_r = 0.316$, and $\kappa = \sqrt{2}\Omega$.

Fig. 16.— The figure shows the dependence of the one-armed ($m = 1$) spiral wave instability on ξ_g for three values of ξ_*. **a** shows the dependence of $(\omega_r)_m$ on ξ_g, where $(\omega_r)_m$ is the value of ω_r at the point where $|\omega_r(\overline{k}_r)|$ is a minimum for $\overline{k}_r$ within the unstable range. **b** shows the dependence of the maximum growth on ξ_g. The figure assumes $Q_* = 1.6$ for both stellar components, $c_s/\sigma_r = 0.5$, and $\kappa = \sqrt{2}\Omega$. The largest wave amplification occurs when $(\omega_r)_m = 0$, and panel **a** shows that this happens for $\xi_* + \xi_g \approx \frac{1}{2}$.

Fig. 17.— Geometry of a one-armed leading spiral shock wave in counter-rotating gas which leads to inward radial motion of the gas.